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The cusp forms of weight 3 on $\Gamma_2(2,4,8)$.
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Let Γ_g be the Siegel modular group:

$$\Gamma_g=\mathrm{Sp}_{2g}(\mathbf{Z})=\left\{M\in M_{2g}(\mathbf{Z})\colon\ MJ_{2g}M^{\mathsf{T}}=J_{2g},\ J_{2g}=\left(\begin{smallmatrix}0&-I_n\\ I_n&0\end{smallmatrix}\right)\right\}.$$

Let $\Gamma_g(n)$ be the subset of Γ_g consisting of the matrices which are $\equiv I \bmod n$,

$$\Gamma_g(4,8)=\left\{\left(\begin{smallmatrix}A&B\\ C&D\end{smallmatrix}\right)\in\Gamma_g(4)\colon\ \mathrm{diag}(B)\equiv\mathrm{diag}(C)\equiv 0 \bmod 8\right\},$$

$$\Gamma_g(2,4,8)=\left\{\left(\begin{smallmatrix}I+4A'&B\\ C&I+4D'\end{smallmatrix}\right)\in\Gamma_g(4,8)\colon\ \mathrm{trace}(A')\equiv 0 \bmod 2\right\}.$$

The Siegel upper half-space of degree g is defined as the set $\mathcal{H}_g$ of complex symmetric $n\times n$ matrices with positive definite imaginary part. Let Γ' be a congruence subgroup of Γ_g and denote by $M_k(\Gamma')$ the vector space of modular form of weight k for Γ'. One defines the Siegel operator Ψ, mapping $f\in M_k(\Gamma')$ to a function on $\mathcal{H}_{g-1}$, by $\Psi(f)(\tau)=\lim_{t\to\infty}f\left(\begin{smallmatrix}\tau&0\\ 0&it\end{smallmatrix}\right)$, $\tau\in\mathcal{H}_{g-1}$. The subspace $S_k(\Gamma')$ of $M_k(\Gamma')$ is defined by

$$S_k(\Gamma')=\{f\in M_k(\Gamma')\colon$$
$$\Psi(\det(C_\tau+D)^{-k}f((A_\tau+B)(C_\tau+D)^{-1}))(\tau)=0,\ \tau\in\mathcal{H}_{g-1},\ M\in\Gamma_g\}$$

and is called the space of cusp forms.

In this paper, by using the combinatorics of theta constants, the space $S_3(\Gamma_g(2,4,8))$ of cusp forms of weight 3 on the congruence subgroup $\Gamma_g(2,4,8)$ of Γ_g is determined and the splitting of $S_3(\Gamma_g(2,4,8))$ into irreducible Γ-representations is determined. Also, the actions of the Hecke algebra on $S_3(\Gamma_g(2,4,8))$ are determined. The eigenspaces and eigenvalues of these operators are studied. The Hecke polynomials for several cusp forms and for some small primes p are described.

In the introduction, the authors state: "Most of the forms we consider appear to be obtained via liftings from modular forms on subgroups of $\mathrm{SL}_2(\mathbf{Z})$. In one case the Hecke polynomials suggest that the modular form is related to a Hecke character of the field of eighth roots of unity. There is one case in which the Hecke polynomials of the cusp form do not allow one of these interpretations. In this paper we do not actually try to prove that most of the forms are indeed liftings."

Reviewed by *YoungJu Choie*